Lecture 7 - January 31

Math Review

Functions, Modelling

Announcement

Lab1 $\xrightarrow{\text{solution}}$ today

Lab2 → nex Monday

WTI

Exercises: Algebraic Properties of Relational Operations

$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$

Define the image of set s on r in terms of other relational operations.

Hint: What range of value should be included?

set of range values

$\boxed{r[s]} = \text{ran}(\boxed{s \lhd r})$

another relation

domain restriction

s should be: $s \subseteq \text{dom}(r)$ otherwise: result is ϕ.

Define r overridden with set t in terms of other relational operations.

Hint: To be in t's domain or not to be in t's domain?

$r \mathbin{\lhd\!\!\!+} t = \boxed{t \cup (\boxed{\text{dom}(t) \lhd r})}$

a relation

another relation

$(\text{dom}(r) \setminus \text{dom}(t)) \lhd r$

Functional Property

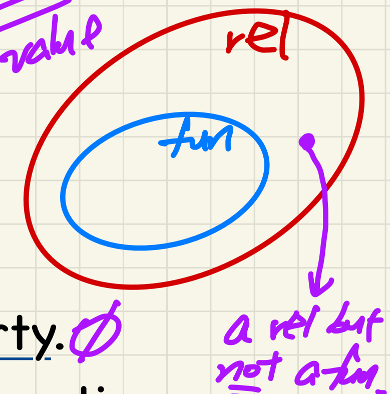
e.g. {(a,1), (b,2)} relation

e.g. $\{\underbrace{(a,1)}_{s\ t1}, (b,2), \underbrace{(a,3)}_{s\ t2}\}$

↳ a relation
not a function!

$$
\boxed{
\begin{array}{l}
\text{isFunctional}(r) \Leftrightarrow \quad r \in S \leftrightarrow T \\
\quad \forall \underbrace{s}_{\in S}, \underbrace{t1, t2}_{\in T} \bullet \\
\quad\quad \underbrace{(s \in S \wedge t1 \in T \wedge t2 \in T)}_{\text{cnt.1}} \\
\quad\quad \Rightarrow \\
\quad\quad (\underbrace{(s, t1) \in r \wedge (s, t2) \in r}_{\text{cnt.2}} \Rightarrow \boxed{t1 = t2})
\end{array}
}
$$

each domain value s maps to ~~at most one~~ range value con.



Q: Smallest relation satisfying the functional property. $\emptyset$

Q: How to **prove** or **disprove** that a relation r is a function.

Q: Rewrite the functional property using **contrapositive**.

Prove r is a fun

(1) for each pair in r, satisfying cnt1, satisfies cnt2 ⇒ con.

(2) Show $r = \emptyset$.

Disprove r is a fun

(1) r is not empty, there's $(s, t1) \in r \wedge (s, t2) \in r$ but $t_1 \neq t2$

$$\boxed{p \Rightarrow q \equiv \neg q \Rightarrow \neg p}$$

isFunctional(r) $\Leftrightarrow$

$\forall$ s, t1, t2 $\bullet$

$(s \in S \wedge t1 \in T \wedge t2 \in T)$

$\Rightarrow$

$\boxed{((s, t1) \in r \wedge (s, t2) \in r \Rightarrow t1 = t2)}$

$\equiv$ contra-positive

$$t1 \neq t2 \Rightarrow \boxed{\neg((s, t1) \in r \wedge (s, t2) \in r)}$$

$$\boxed{(s, t1) \notin r \vee (s, t2) \notin r}$$

If t1 and t2 are distinct values, then we cannot have s mapping to both of them.

Partial Functions vs. Total Functions

$\not\rightarrow$ $+\!\!\rightarrow$

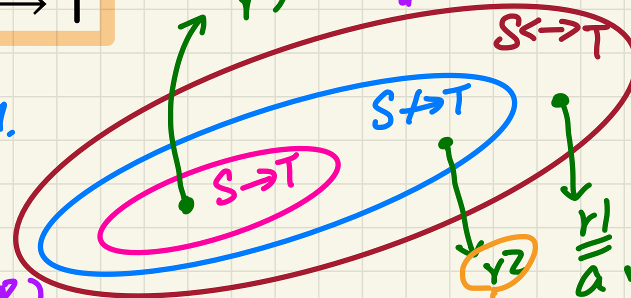
partial

$$r \in S \boxed{\nrightarrow} T \Leftrightarrow (\text{isFunctional}(r) \land \text{dom}(r) \subseteq S)$$

$$r \in S \rightarrow T \Leftrightarrow (\text{isFunctional}(r) \land \text{dom}(r) = S)$$

total

Exercise: Visualize $S \nrightarrow T$ vs. $S \longrightarrow T$



Every function is a partial function.

1. a relation ($\because$ set of pairs)
2. a partial fun ($\because$ not violate fun. prop.)
3. a total fun ($\because$ dom $= S$)

e.g., $\{ \{(2, a), (1, b)\}, \{(2, a), (3, a), (1, b)\} \} \subseteq \{1, 2, 3\} \nrightarrow \{a, b\}$

a function but dom(r) ⊂ S

e.g., $\boxed{\{(2, a), (3, a), (1, b)\}} \in \{1, 2, 3\} \rightarrow \{a, b\}$

e.g., $\boxed{\{(2, a), (1, b)\}} \notin \{1, 2, 3\} \rightarrow \{a, b\}$

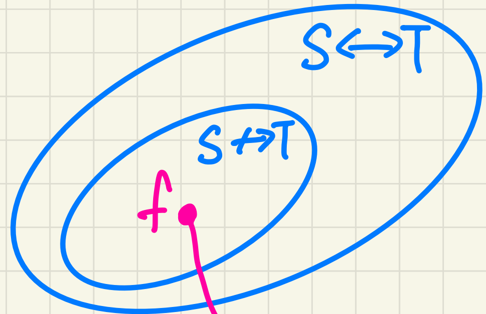
1. a rel.
2. a partial fun.
3. not a total fun. $\because$ $\underbrace{\text{dom}}_{\{2,1\}} \neq \{1,2,3\}$

e.g., $\{(2, a), (1, b), (3, a), (1, a)\} \notin \{1, 2, 3\} \rightarrow \{a, b\}$

↳ 1. a rel. 2. not a fun.

Relational Image vs. Functional Application

A function is a relation.



1. a relation f[s]
2. a function

$f \in \overset{S}{\boxed{\{1, 2, 3\}}} \text{ ⇸ } \{a, b\}$

$f = \{ (3, a), (1, b) \}$

↳ a rel, a partial fun, <u>not</u> a total fun.

Exercises:

$f[\{3\}] = \{a\}$ — $f(3) = a$

$f[\{1\}] = \{b\}$ — $f(1) = b$

$\boxed{f[\{2\}] = \emptyset}$ → for a function that's partial but <u>not</u> total (i.e. $dom(f) \subset S$, there's at least one value in S that maps to nothing in f).

$f(2) = \text{undefined}$ — $\perp$ ⇝ bottom

↓ input: singleton sets

Modelling Decision: Relations vs. Functions

An organization has a system for keeping **track** of its employees as to where they are on the premises (e.g., ``Zone A, Floor 23''). To achieve this, each employee is issued with an active badge which, when scanned, synchronizes their current positions to a central database.

Assume the following two sets:

- *Employee* denotes the **set** of all employees working for the organization.
- *Location* denotes the **set** of all valid locations in the organization.

Is where_is ∈ Employee <-> Location appropriate? ✗

→ {("alan", LSB106), ("alan", VC102)}

Is where_is ∈ Employee → Location appropriate?

dom(where_is) = Employee ✗ not realistic to expect all employees to be present in company all the time

Is where_is ∈ Employee ⇸ Location appropriate?

↳ a relation satisfying the fun. prop. but is not total

Functions

	dom injective	ran surjective	dom, ran bijective
partial	⤔	⤀	n.a.
total	$\rightarrowtail$	$\twoheadrightarrow$	⤖